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$B \to K$ transition form factor up to $\mathcal{O}\left(1/m_b^2\right)$ within the k_{T} factorization approach

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Abstract. We apply the k_{T} factorization approach to deal with the $B \to K$ transition form factor $F_{+,0}^{B\to K}(q^2)$ in the large recoil regions. The B-meson wave functions Ψ_B and $\bar{\Psi}_B$ that include the 3-particle Fock states' contributions are adopted to give a consistent PQCD analysis of the form factor up to $\mathcal{O}\left(1/m_b^2\right)$. It has been found that the two wave functions Ψ_B and $\bar{\Psi}_B$ can give sizable contributions to the form factor and should be kept for a better understanding of the B-meson decays. Next the contributions from different twist structures of the kaon wave function are discussed, including $\mathrm{SU}_f(3)$-breaking effects. A sizable contribution from the twist-3 wave function Ψ_p is found, whose model dependence is discussed by taking two groups of parameters that are determined by different distribution amplitude moments obtained in the literature. It is also shown that $F_{+,0}^{B\to K}(0) = 0.30 \pm 0.04$ and $\left[F_{+,0}^{B\to K}(0)/F_{+,0}^{B\to \pi}(0)\right] = 1.13 \pm 0.02$, which are more reasonable and consistent with the light-cone sum rule results in the large recoil regions.

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1 Introduction

A study of the heavy-to-light exclusive processes plays a complementary role in the determination of the fundamental parameters of the standard model and in developing the QCD theory. Also, there is an increasing demand for more reliable QCD calculations of the heavy-to-light form factors. We have done a consistent analysis of the $B \to \pi$ transition form factor in [1, 2], which shows that the results from the PQCD approach, the lattice QCD approach and the QCD light-cone sum rules (LCSRs) are complementary, and by combining the results of those three approaches one can obtain an understanding of the $B \to \pi$ transition form factor in the whole physical region. It is argued that by applying the k_{T} factorization approach [3–8], where the transverse momentum dependence for both the hard-scattering part and the nonperturbative wave function, the Sudakov effects and the threshold effects are included, one can regulate the endpoint singularity from the hard-scattering part effectively and derive a more reliable PQCD result for the B-meson decays. Furthermore, by applying the B-meson wave functions up to next-to-leading Fock state [2], we calculated the $B \to \pi$ transition form factor up to $\mathcal{O}\left(1/m_b^2\right)$ and also discussed the reasonable regions for the two phenomenological B-meson wave function parameters $\bar{\Lambda}$ and δ, where $\bar{\Lambda}$ is the effective mass of the B-meson, which determines the B-meson's leading Fock state behavior, and δ is a typical parameter that determines the broadness of the B-meson transverse distribution. Since pion and kaon are pseudoscalar mesons, it will be interesting to give a consistent PQCD analysis of the $B \to K$ transition form factor up to order $\mathcal{O}\left(1/m_b^2\right)$ based on the results of the $B \to \pi$ transition form factor.

In the literature, the $B \to K$ transition form factor has been studied in several approaches [9–14]. A PQCD calculation has been done in [10], which can be roughly treated as a leading-order estimation, $\mathcal{O}(1/m_b)$, since some of the power suppressed terms both in the hard-scattering amplitude and the B-meson wave function have been neglected. The $B \to K$ transition form factor has also been analyzed by several groups in the QCD LCSR approach [11–14], where some extra treatments of the correlation function either on the B-meson side or on the kaonic side are adopted to improve their LCSR estimations. A new sum rule for the $B \to K$ form factor is derived by expanding the correlation function near the light cone in terms of the B-meson distributions [11], in which the contributions of the quark–antiquark and quark–antiquark–gluon components in the B-meson are taken into account. In [12] the improved LCSR approach that had been presented in [15, 16] was adopted to eliminate the contributions from the most uncertain kaonic twist-3 wave functions and to enhance the reliability of sum rule calculations of the $B_s \to K$ form fac-

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tor. A systematic QCD LCSR calculation has been done in [13] by including the one-loop radiative corrections to the kaonic twist-2 and twist-3 contributions and the kaonic leading-order twist-4 corrections. It can be found that the main uncertainties in the estimation of the $B \to K$ transition form factor come from the B-meson wave function and the kaonic twist-2 and twist-3 wave functions.

In doing the PQCD calculations on the B-meson decays, an important issue is whether we need to take the two wave functions Ψ_B and $\bar{\Psi}_B$ into consideration, or is Ψ_B simply enough? By taking the frequently used first type definition of $\Psi_B = \frac{\Psi_B^+ + \Psi_B^-}{2}$ and $\bar{\Psi}_B = \frac{\Psi_B^+ - \Psi_B^-}{2}$, where $\Psi_B^\pm$ are defined in [17], it can be found that [18, 19] their distribution amplitudes have quite different endpoint behaviors even in the Wandzura–Wilczek (WW) approximation [20]; such a difference may be strongly enhanced by the hard-scattering kernel. For example, the ratio of the contributions of $\bar{\Psi}_B$ and Ψ_B is about (-70%) [1, 21] for the $B \to \pi$ form factor in the large recoil regions. So the contribution from $\bar{\Psi}_B$ in the above definition cannot be neglected, and it is needed to suppress the big contribution coming from Ψ_B so as to obtain reasonable total contributions. To derive a more accurate estimation, [10] presented the second type of definition of $\Psi_B = \Psi_B^+$ and $\bar{\Psi}_B = \left(\Psi_B^+ - \Psi_B^-\right)$, under which the contribution from $\bar{\Psi}_B$ is of order $\mathcal{O}(1/m_b)$ compared to that of Ψ_B [22]. For convenience, in the following, we shall adopt the second type definition of Ψ_B and $\bar{\Psi}_B$ to do our calculation. Then one may ask: is it enough to give a $\mathcal{O}\left(1/m_b^2\right)$ estimation with Ψ_B and $\bar{\Psi}_B$ in the WW approximation? As has been pointed out in [23], the 3-particle Fock states' contributions to the B-meson wave function can be estimated by attaching an extra gluon to the internal off-shell quark line, and then $(1/m_b)$ power suppression is induced in comparison to that of the WW-part B-meson wave functions. Recently, the B-meson light-cone wave functions have been investigated up to next-to-leading order in a Fock state expansion in the heavy-quark limit [2]. It was shown that by using the relations between 2- and 3-particle wave functions derived from the QCD equations of motion and the heavy-quark symmetry, one can give a constraint on the transverse momentum dependence of the B-meson wave function, whose distribution tends to a hyperbola-like curve, different from the simple delta function that is derived in the WW approximation. These results provide us with an opportunity to give a consistent PQCD analysis of the $B \to K$ form factor up to order $\mathcal{O}\left(1/m_b^2\right)$.

Another issue we need to be more careful about is the kaonic wave functions. The distribution amplitude (DA) for the twist-2 wave function Ψ_K has been thoroughly studied, e.g. by the light-front quark model [24, 25], the LCSR approach [26–29] and lattice calculations [30, 31], etc. In [26], the QCD sum rule for the diagonal correlation function of local and nonlocal axial-vector currents is used, in which the contributions of condensates up to dimension six and the $\mathcal{O}(\alpha_\mathrm{s})$ corrections to the quark-condensate term are taken into account. The first Gegenbauer moment $a_1^K(1\,\mathrm{GeV})$ of the twist-2 DA derived there, i.e. $a_1^K(1\,\mathrm{GeV}) = 0.05 \pm 0.02$, is consistent with that of the lattice calculations [30, 31], so we shall constrain $a_1^K(1\,\mathrm{GeV})$ within this range when constructing a model for Ψ_K. As for the twist-3 wave function Ψ_p, the calculation of it has more uncertainty than that for the leading twist, e.g. its DA moments in [29, 32, 33] are quite different from each other, where the DA moments in [29, 32] are derived by using the QCD light-cone sum rules and the moments in [33] are derived based on the effective chiral action from the instanton. Under the PQCD approach, according to our experience on the $B \to \pi$ transition form factor [1] and the pion electromagnetic form factor [34, 35], it can be found that for a twist-3 wave function with a better endpoint behavior other than the asymptotic one, the twist-3 contributions are indeed power suppressed compared to the leading twist's contribution, which favors the conventional power counting rules. In the present paper, we shall adopt two groups of DA moments [29, 33] together with the Brodsky–Huang–Lepage (BHL) prescription [36–38] to construct a model for Ψ_p, and then we discuss its uncertainty for the $B \to K$ transition form factor. The $\mathrm{SU}_f(3)$-breaking effects shall also be included for constructing the kaonic wave functions.

The purpose of the paper is to reexamine the $B \to K$ transition form factor in the PQCD k_T factorization approach up to $\mathcal{O}\left(1/m_b^2\right)$. In the k_T factorization approach, the full transverse momentum dependence (k_T dependence) for both the hard-scattering part and the nonperturbative wave function, the Sudakov effects and the threshold effects are included to cure the endpoint singularity. Furthermore, we shall analyze the power suppressed contributions from both the wave functions and the hard scattering amplitude and then give a consistent analysis of the form factor up to $\mathcal{O}\left(1/m_b^2\right)$, which have not been considered in the literature. In Sect. 2, we give the calculational technology for the form factor in the large recoil regions. Also we present the model wave functions of the kaon with better endpoint behavior in this section, which are constructed based on the BHL prescription [36–38] and the DA moments obtained in [29, 33]. In Sect. 3, we give our numerical results. Our conclusion and a brief summary are presented in the final section.

2 Calculational technology for the $B \to K$ transition form factor

The $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ are defined as follows:

$$\begin{aligned}
&\left\langle K(P_K)\,|\bar{s}\gamma_\mu b|\,\bar{B}(P_B)\right\rangle \\
&\quad = \left[(P_B + P_K)_\mu - \frac{M_B^2 - M_K^2}{q^2} q_\mu\right] F_+^{B\to K}\left(q^2\right) \\
&\qquad + \frac{M_B^2 - M_K^2}{q^2} q_\mu F_0^{B\to K}\left(q^2\right),
\end{aligned} \tag{1}$$

where $F_+^{B\to K}(0)$ should be equal to $F_0^{B\to K}(0)$ so as to cancel the poles at $q^2 = 0$. The amplitude for the $B \to K$ transition form factor can be factorized into the convolution of the wave functions for the respective hadrons with the hard-scattering amplitude. In the large recoil regions, the form factor of the $B \to K$ transition is dominated by single gluon exchange in the lowest order. In [1], we have done a consistent analysis of the $B \to \pi$ transition form factor within the k_T factorization approach, where the power suppressed terms up to $\mathcal{O}\left(1/m_b^2\right)$ have been kept explicitly in the hard-scattering amplitude. The interested reader may refer to [1] for more details[1]. More specifically, for the present case, one needs to know the momentum projection for the matrix element of the kaon and B-meson in deriving the hard-scattering amplitude. By keeping the transverse momentum dependence in the wave function, the momentum projection for the matrix element of the kaon has the following form:

$$\begin{aligned} M^K_{\alpha\beta} &= \frac{\mathrm{i}f_\pi}{4}\left\{ \not{p}\gamma_5 \Psi_K\left(x,\mathbf{k}_\perp\right) - \mu_K\gamma_5\left(\Psi_p\left(x,\mathbf{k}_\perp\right)\right.\right. \\ &\left.\left.\left. -\mathrm{i}\sigma_{\mu\nu}\left(n^\mu\bar{n}^\nu\frac{\Psi'_\sigma\left(x,\mathbf{k}_\perp\right)}{6} - p^\mu\frac{\Psi_\sigma\left(x,\mathbf{k}_\perp\right)}{6}\frac{\partial}{\partial\mathbf{k}_{\perp\nu}}\right)\right)\right\}_{\alpha\beta}, \end{aligned} \tag{2}$$

where f_K is the kaon decay constant and μ_K is the phenomenological parameter $\mu_K = M_K^2/(m_s+m_u)$, which is a scale characterized by chiral perturbation theory. $\Psi_K(x,\mathbf{k}_\perp)$ is the twist-2 wave function, and $\Psi_p(x,\mathbf{k}_\perp)$ and $\Psi_\sigma(x,\mathbf{k}_\perp)$ are twist-3 wave functions, respectively. $\Psi'_\sigma(x,\mathbf{k}_\perp) = \partial\Psi_\sigma(x,\mathbf{k}_\perp)/\partial x$, $\mathbf{n} = (\sqrt{2},0,\mathbf{0_1})$ and $\bar{\mathbf{n}} = (0,\sqrt{2},\mathbf{0_1})$ are two null vectors that point to the plus and the minus directions, respectively. Also the momentum projection for the matrix element of the B-meson can be written as [21,39]

$$\begin{aligned} M^B_{\alpha\beta} = -\frac{\mathrm{i}f_B}{4}\left\{\frac{\not{p}_B + M_B}{2}\left[\not{n}\Psi_B^+\left(\xi,\mathbf{l_1}\right) + \not{\bar{n}}\Psi_B^-\left(\xi,\mathbf{l_1}\right)\right.\right. \\ \left.\left. -\Delta(\xi,\mathbf{l_1})\gamma^\mu\frac{\partial}{\partial\mathbf{l}_\perp^\mu}\right]\gamma_5\right\}_{\alpha\beta}, \end{aligned} \tag{3}$$

where $\xi = \frac{l^+}{M_B}$ is the momentum fraction for the light spectator quark in the B-meson, and $\Delta(\xi,\mathbf{l_1}) = M_B\int_0^\xi \mathrm{d}\xi'(\Psi_B^-(\xi',\mathbf{l_1}) - \Psi_B^+(\xi',\mathbf{l_1}))$. The four-component $\mathbf{l}_\perp^\mu$ in (3) is defined by $\mathbf{l}_\perp^\mu = l^\mu - \frac{(l^+n^\mu + l^-\bar{n}^\mu)}{2}$ with $\mathbf{l} = \left(\frac{l^+}{\sqrt{2}},\frac{l^-}{\sqrt{2}},\mathbf{l}_\perp\right)$. By including the Sudakov form factors and the threshold resummation effects, one can obtain the formulae for the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ in the transverse configuration b-space, which can be simply obtained from [1] by changing the pion wave functions to the present case of kaons and by changing Ψ_B and $\bar{\Psi}_B$ to the second type of definition as described in the introduction.

In the PQCD approach, the transverse momenta $\mathbf{k}_\perp$ of the parton are not negligible around the endpoint region. The relevant Sudakov factors from both $k_\perp$ and the threshold resummation [40–43] can cure the endpoint singularity, which makes the calculation of the hard amplitudes infrared safe, and then the main contribution comes from the perturbative region. Also it is necessary to keep the transverse momentum dependence in the wave functions to derive a more reliable estimation in PQCD. In principle, the Bethe–Salpeter formalism [44] and the discretized light-cone quantization approach [45–48] could determine the hadronic wave functions, but in practice there are many difficulties in getting the exact wave functions at present. The BHL prescription [36–38], which connects the equal-time wave function in the rest frame and the wave function in the infinite momentum frame, provides a useful way to use the approximate bound state solution of a hadron in terms of the quark model as the starting point for modeling the hadronic wave function. So in the present paper, we will adopt the BHL prescription for constructing the kaonic wave functions. For the B-meson wave function, these have been investigated up to next-to-leading order in the Fock state expansion in the heavy-quark limit in [2], which shall be adopted in our discussions.

A simple model has been presented in [2] for the B-meson wave functions Ψ_B^+ and Ψ_B^-, which keep the main features caused by the 3-particle Fock states and whose transverse momentum dependence is still the delta-like function of the off-shell energy of the valence quarks, but it shall broaden the transverse momentum dependence in the WW approximation to a certain degree. In compact parameter b_B-space, it reads [2]

$$\begin{aligned} &\Psi_B^+\left(\xi,b_B\right) \\ &= (16\pi^3)\frac{M_B^2\xi}{\omega_0^2}\exp\left(-\frac{M_B\xi}{\omega_0}\right) \\ &\quad\times\left(\Gamma[\delta]J_{\delta-1}[\kappa] + (1-\delta)\Gamma[2-\delta]J_{1-\delta}[\kappa]\right)\left(\frac{\kappa}{2}\right)^{1-\delta} \end{aligned} \tag{4}$$

and

$$\begin{aligned} &\Psi_B^-\left(\xi,b_B\right) \\ &= \left(16\pi^3\right)\frac{M_B}{\omega_0}\exp\left(-\frac{M_B\xi}{\omega_0}\right) \\ &\quad\times\left(\Gamma[\delta]J_{\delta-1}[\kappa] + (1-\delta)\Gamma[2-\delta]J_{1-\delta}[\kappa]\right)\left(\frac{\kappa}{2}\right)^{1-\delta}, \end{aligned} \tag{5}$$

where $\omega_0 = 2\bar{\Lambda}/3$, $\bar{\xi} = \bar{\Lambda}/M_B$ and $\kappa = \theta(2\bar{\xi}-\xi)\sqrt{\xi(2\bar{\xi}-\xi)}M_B b_B$. The factor $(16\pi^3)$ is introduced to ensure that their Fourier transformation, i.e. $\Psi_B^\pm(\xi,\mathbf{k}_\perp)$, satisfy the normalization, $\int\frac{\mathrm{d}\xi\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\Psi_B^\pm(\xi,\mathbf{k}_\perp) = 1$. It can be found that Ψ_B^+ and Ψ_B^- have the same transverse momentum dependence and only the two phenomenological parameters $\bar{\Lambda}$ and δ are introduced. $\bar{\Lambda}$ is the effective mass of the B-meson, $\bar{\Lambda} = M_B - m_b$; this determines the B-meson's leading Fock state behavior. δ is a typical parameter that determines the broadness of the B-meson transverse distribution in comparison to the WW-like one.

[1] Three typo errors are found in [1], i.e. in (3) $\frac{\not{p}_B M_B}{2}$ should be changed to $\frac{\not{p}_B + M_B}{2}$, in (5) the factor $[3-\eta-x\eta]$ should be changed to $[3-\eta+x\eta]$ and in (7) y should be changed to η.

The WW-like B-meson wave functions in the compact parameter b_B-space can be found in [1]. A direct comparison shows that when $\delta \to 1$, the transverse momentum dependence of the B-meson wave function in (4) and (5) returns to a simple δ-function, which is the same as that of the B-meson wave function in the Wandzura–Wilczek approximation [18, 49, 50]. According to the definitions, we have $\Psi_B(\xi, b_B) = \Psi_B^+(\xi, b_B)$, $\bar{\Psi}_B(\xi, b_B) = \Psi_B^+(\xi, b_B) - \Psi_B^-(\xi, b_B)$ and $\Delta(\xi, b_B) = -M_B \int_0^\xi \mathrm{d}\xi' \bar{\Psi}_B(\xi', b_B)$.

Next, we construct the kaonic twist-2 wave function based on its first Gegenbauer moment a_1^K and on the BHL prescription [36–38]. The first Gegenbauer moment a_1^K has been studied by the light-front quark model [24, 25], the LCSR approach [26, 27, 29], lattice calculations [30, 31], etc. In [26], a QCD sum rule for the diagonal correlation function of local and nonlocal axial-vector currents is used in which the contributions of condensates up to dimension six and $\mathcal{O}(\alpha_{\mathrm{s}})$ corrections to the quark-condensate term are taken into account. The moments derived there are close to that of the lattice calculation [30, 31], so we shall take $a_1^K(1\,\mathrm{GeV}) = 0.05 \pm 0.02$ to determine the model wave function Ψ_K. Based on the BHL prescription, we take the twist-2 wave function of the kaon as

$$\begin{aligned}\Psi_K(x, \mathbf{k}_\perp) &= \left[1 + B_K C_1^{3/2}(2x-1)\right] \\ &\times \frac{A_K}{x(1-x)} \exp\left[-\beta_K^2 \left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_s^2}{1-x}\right)\right],\end{aligned} \tag{6}$$

where $q = u, d$, and $C_1^{3/2}(1-2x)$ is the Gegenbauer polynomial. In comparison to the pion wave function (see, e.g., [51]), it can be found that the $\mathrm{SU}_f(3)$ symmetry is broken by a non-zero B_K and by the mass difference between the s-quark and the u- (or d-) quark in the exponential factor. The $\mathrm{SU}_f(3)$ symmetry breaking in the lepton decays of heavy pseudoscalar mesons and in the semileptonic decays of mesons have been studied in [52, 53]. For definiteness, we take the conventional values for the constituent quark masses: $m_q = 0.30\,\mathrm{GeV}$ and $m_s = 0.45\,\mathrm{GeV}$. The parameters A_K, B_K and β_K can be determined by the value of a_1^K together with the normalization condition

$$\int_0^1 \mathrm{d}x \int_{k_\perp^2 < \mu_0^2} \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \Psi_K(x, \mathbf{k}_\perp) = 1 \tag{7}$$

and the constraint $\langle \mathbf{k}_\perp^2 \rangle_K^{1/2} \approx \langle \mathbf{k}_\perp^2 \rangle_\pi^{1/2} = 0.350\,\mathrm{GeV}$ [54], where the average value of the transverse momentum square is defined as

$$\langle \mathbf{k}_\perp^2 \rangle_K^{1/2} = \frac{\int \mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp \left|\mathbf{k}_\perp^2\right| |\Psi_K(x, \mathbf{k}_\perp)|^2}{\int \mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp |\Psi_K(x, \mathbf{k}_\perp)|^2}.$$

The parameter μ_0 in the model wave function stands for some hadronic scale that is of order $\mathcal{O}(1\,\mathrm{GeV})$. For clarity, we set $\mu_0 = 1\,\mathrm{GeV}$. The DA $\phi_K(x, \mu_0)$ is defined as $\phi_K(x, \mu_0) = \int_{k_\perp^2 < \mu_0^2} \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \Psi_K(x, \mathbf{k}_\perp)$. The first Gegenbauer moment $a_1^K(\mu_0)$ of [26, 27, 29] can be defined by

$$a_1^K(\mu_0) = \frac{\int_0^1 \mathrm{d}x \phi_K(1-x, \mu_0) C_1^{3/2}(2x-1)}{\int_0^1 \mathrm{d}x 6x(1-x) \left[C_1^{3/2}(2x-1)\right]^2}, \tag{8}$$

where $\phi_K(1-x, \mu_0)$, different from $\phi_K(x, \mu_0)$, should be adopted, since in [26, 27, 29] x stands for the momentum fraction of the s-quark in the kaon ($\bar{K}$), while in the present paper we take x as the momentum fraction of the light q, the (anti-) quark, in the kaon $(K)^2$. Based on the above discussions, we can obtain the values for A_K, B_K and β_K:

$$\begin{aligned}&A_K \cong 2.71 \times 10^2\,\mathrm{GeV}^{-1}, \quad B_K \cong \left[0.116 - 0.9 a_1^K(\mu_0)\right], \\ &\beta_K \cong 0.877\,\mathrm{GeV}^{-1},\end{aligned} \tag{9}$$

where the values of A_K and β_K are almost constant, i.e. their changes $(\delta A_K/A_K)$ and $(\delta\beta_K/\beta_K)$ are less than 0.001 by varying $a_1^K(\mu_0)$ within the range of $[0.03, 0.07]$. More specifically, for the case of $a_1^K(\mu_0) = 0.05$, we have

$$\begin{aligned}&A_K = 2.71 \times 10^2\,\mathrm{GeV}^{-1}, \quad B_K = 0.071, \\ &\beta_K = 0.877\,\mathrm{GeV}^{-1}.\end{aligned}$$

As will be seen, the contributions from the twist-3 wave function $\Psi_\sigma\left(x, \vec{k_\perp}\right)$ are less important than those of $\Psi_K\left(x, \vec{k_\perp}\right)$ and $\Psi_p\left(x, \vec{k_\perp}\right)$, which is similar to the case of the form factor of the $B \to \pi$ transition [1]. So, based on the BHL prescription, we directly take the twist-3 wave function Ψ_σ of the kaon as

$$\Psi_\sigma(x, \mathbf{k}_\perp) = A_\sigma \exp\left[-\beta_K^2 \left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_s^2}{1-x}\right)\right], \tag{10}$$

where A_σ can be determined by its normalization condition, i.e. $A_\sigma = 1.36 \times 10^3\,\mathrm{GeV}^{-1}$.

As for the twist-3 wave function $\Psi_p(x, \vec{k}_\perp)$, its DA asymptotic behavior is $\phi_p^{as}(x, \infty) = 1$, so its endpoint singularity is much more serious. Now the transverse momentum dependence of $\Psi_p(x, \vec{k}_\perp)$ is much more important than that of $\Psi_K(x, \vec{k}_\perp)$ and $\Psi_\sigma(x, \vec{k}_\perp)$ for curing the endpoint singularity. One can construct $\Psi_p(x, \vec{k}_\perp)$ in the following form:

$$\begin{aligned}\Psi_p\left(x, \vec{k}_\perp\right) &= \left[1 + B_p C_1^{1/2}(2x-1) + C_p C_2^{1/2}(2x-1)\right] \\ &\times \frac{A_p}{x(1-x)} \exp\left[-\beta_K^2 \left(\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_s^2}{1-x}\right)\right],\end{aligned} \tag{11}$$

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[2] In the literature, there are some ambiguities in the use of $\phi_{K,p}(x, \mu_0)$ or $\phi_{K,p}(1-x, \mu_0)$ in connection to the hard-scattering part. This will cause errors when the $\mathrm{SU}_f(3)$ symmetry is broken.

where x stands for the light quark's momentum fraction, $C_1^{1/2}(2x-1)$ and $C_2^{1/2}(2x-1)$ are Gegenbauer polynomials, and the coefficients A_p, B_p and C_p can be determined by its DA moments. The DA $\phi_p(x,\mu_0)$ is defined as $\phi_p(x,\mu_0) = \int_{k_\perp^2<\mu_0^2} \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \Psi_p(x,\mathbf{k}_\perp)$.

To discuss the uncertainty caused by Ψ_p, we take two groups of DA moments that have been obtained in [29, 33] to determine the coefficients A_p, B_p and C_p, where the moments in [29] are derived by using the QCD light-cone sum rules and the moments in [33] are derived based on the effective chiral action from the instanton:

$$\text{Group 1 [26]}: \ \langle x^0\rangle_p^K = 1\,, \quad \langle x^1\rangle_p^K = 0.06124\,, \quad \langle x^2\rangle_p^K = 0.36757\,, \tag{12}$$

$$\text{Group 2 [30]}: \ \langle x^0\rangle_p^K = 1\,, \quad \langle x^1\rangle_p^K = 0.00678\,, \quad \langle x^2\rangle_p^K = 0.35162\,. \tag{13}$$

Here the moments are defined as $\langle x^i\rangle_p^K = \int_0^1 \mathrm{d}x\,(2x-1)^i \phi_p(1-x,\mu_0)$ with $i=(0,1,2)$. It should be noted that the moments defined in [29, 33] are for $\phi_p(1-x,\mu_0)$, different from $\phi_p(x,\mu_0)$, since in these references x stands for the momentum fraction of the s-quark in the kaon ($\bar{K}$), while in the present paper x stands for the momentum fraction of the light quark q in the kaon (K). Taking the above two groups of DA moments for ϕ_p, the parameters of $\Psi_p(x,\vec{k_\perp})$ can be determined:

$$\text{Group 1}: \ A_p^1 = 382\,\mathrm{GeV}^{-1}\,, \quad B_p^1 = 0.311\,, \quad C_p^1 = 1.61\,, \tag{14}$$

$$\text{Group 2}: \ A_p^2 = 422\,\mathrm{GeV}^{-1}\,, \quad B_p^2 = 0.257\,, \quad C_p^2 = 1.52\,. \tag{15}$$

The distribution amplitudes for these two groups of parameters are shown in Fig. 1, where $\phi_p^1(x,\mu_0)$ is determined by the Group 1 parameters and $\phi_p^2(x,\mu_0)$ is determined by the Group 2 parameters respectively. For comparison, we also draw the distributions derived in [29, 33] in Fig. 1, i.e. $\phi_p^{\mathrm{sr}}(x,\mu_0)$ stands for the DA obtained in [29] and $\phi_p^{\mathrm{in}}(x,\mu_0)$ stands for that of [33]. One may observe that, different from $\phi_p^{\mathrm{sr}}(x,\mu_0)$ and $\phi_p^{\mathrm{in}}(x,\mu_0)$, both $\phi_p^1(x,\mu_0)$ and $\phi_p^2(x,\mu_0)$ are double humped curves and are highly suppressed in the endpoint region. Such a feature is necessary to suppress the endpoint singularity coming from the hard-scattering kernel and then to derive more reasonable results for the twist-3 contributions to the $B \to K$ form factor.

It is more convenient to transform the kaon wave functions in the compact parameter b_K-space, which can be done with the help of the Fourier transformation,

$$\Psi(x,b_K) = \int_{|\mathbf{k}|<1/b_K} \mathrm{d}^2\mathbf{k}_\perp \exp\left(-\mathrm{i}\mathbf{k}_\perp\cdot\mathbf{b}_K\right)\Psi\left(x,\mathbf{k}_\perp\right),$$

where Ψ stands for Ψ_K, Ψ_p and Ψ_σ, respectively. The upper limit of the integration $|\mathbf{k}_\perp| < 1/b_K$ is necessary to ensure that the wave function is soft enough [55, 56]. After doing the Fourier transformation, we obtain the kaonic

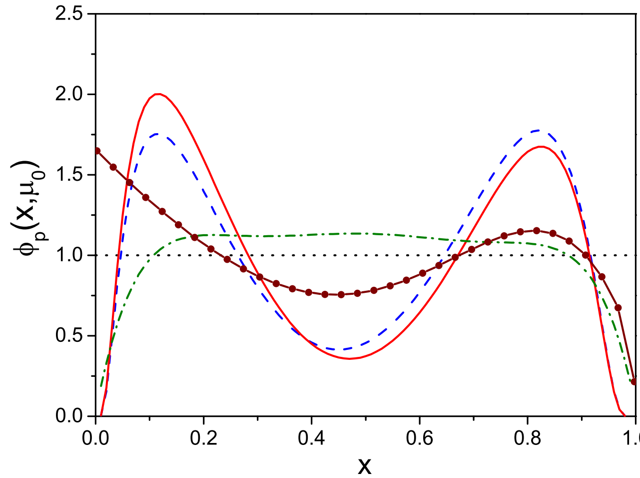


Fig. 1. $\phi_p(x,\mu_0)$ of the kaon with its parameters determined by the two groups of DA moments [29, 33]. The *solid line* and the *dashed line* are for $\phi_p^1(x,\mu_0)$ and $\phi_p^2(x,\mu_0)$ respectively. For comparison, the *big dotted line* and the *dash-dot line* are for $\phi_p^{\mathrm{sr}}(x,\mu_0)$ [29] and $\phi_p^{\mathrm{in}}(x,\mu_0)$ [33] respectively. The *dotted line* is the asymptotic behavior of $\phi_p^{\mathrm{as}}(x,\infty) = 1$

wave functions in the compact parameter b_K-space:

$$\begin{aligned}\Psi_K(x,b_K) &= \frac{2\pi A_K}{x(1-x)}\left[1+B_K C_1^{3/2}(2x-1)\right]\\ &\quad\times\exp\left[-\beta_K^2\left(\frac{m_s^2}{1-x}+\frac{m_q^2}{x}\right)\right]\\ &\quad\times\int_0^{1/b_K}\exp\left(\frac{-\beta_K^2 k_\perp^2}{x(1-x)}\right) J_0(b_K\mathbf{k}_\perp)\mathbf{k}_\perp\mathrm{d}\mathbf{k}_\perp\,,\\ \Psi_\sigma(x,b_K) &= 2\pi A_\sigma\exp\left[-\beta_K^2\left(\frac{m_s^2}{1-x}+\frac{m_q^2}{x}\right)\right]\\ &\quad\times\int_0^{1/b_K}\exp\left(\frac{-\beta_K^2\mathbf{k}_\perp^2}{x(1-x)}\right) J_0(b_K\mathbf{k}_\perp)\mathbf{k}_\perp\mathrm{d}\mathbf{k}_\perp\end{aligned}$$

and

$$\begin{aligned}&\Psi_p(x,b_K)\\ &\quad= \frac{2\pi A_p}{x(1-x)}\left[1+B_p C_1^{1/2}(2x-1)+C_p C_2^{1/2}(2x-1)\right]\\ &\quad\times\exp\left[-\beta_K^2\left(\frac{m_s^2}{1-x}+\frac{m_q^2}{x}\right)\right]\\ &\quad\times\int_0^{1/b_K}\exp\left(\frac{-\beta_K^2\mathbf{k}_\perp^2}{x(1-x)}\right) J_0\left(b_K\mathbf{k}_\perp\right)\mathbf{k}_\perp\mathrm{d}\mathbf{k}_\perp\,.\end{aligned}$$

3 Numerical calculations

In the numerical calculations, we adopt

$$\Lambda_{\overline{MS}}^{(n_f=4)} = 250\,\mathrm{MeV}\,, \quad f_B = 190\,\mathrm{MeV}\,, \quad M_B = 5.279\,\mathrm{GeV}\,,$$
$$f_K = 160\,\mathrm{MeV}\,, \quad M_K = 494\,\mathrm{MeV}\,.$$

For the phenomenological parameter $\mu_K = M_K^2/(m_s + m_u)$, which is a scale characterized by the chiral perturbation theory, we take its value to be $\mu_K \simeq 1.70\,\mathrm{GeV}$.

In the following, we first discuss the properties of $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ that are calculated up to $\mathcal{O}\left(1/m_b^2\right)$, i.e. we show how $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ are affected by the B-meson wave function and the kaonic wave functions. The B-meson wave functions Ψ_B and $\bar{\Psi}_B$ up to next-to-leading order in a Fock state expansion depend on the two phenomenological parameters $\bar{\Lambda}$ and δ. An estimate of $\bar{\Lambda}$ using the QCD sum rule approach gives $\bar{\Lambda} = 0.57 \pm 0.07\,\mathrm{GeV}$ [57]. By comparing the PQCD results of the $B \to \pi$ form factor with the QCD LCSR results and the lattice QCD calculations, [1] shows that $\bar{\Lambda} = 0.55 \pm 0.05\,\mathrm{GeV}$. As for the value of δ, it has been pointed out that if the contribution from the B-meson 3-particle wave function is limited to be within $\pm 20\%$ of that of the WW-like wave function within the energy region of $Q^2 \in [0, \sim 10\,\mathrm{GeV}^2]$, then the value of δ should be restricted within the region of $[0.25, 0.30]$ [2]. For clarity, we take the same regions as obtained from the $B \to \pi$ case [1, 2] for both $\bar{\Lambda}$ and δ, i.e. $\bar{\Lambda} \in [0.50, 0.60]\,\mathrm{GeV}$ and $\delta \in [0.25, 0.30]$, to study the form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ in the large and intermediate energy regions. Furthermore, according to the discussion in the previous section, the remaining uncertainty of the kaonic twist-2 wave function Ψ_K is caused by the value of $a_1^K(1\,\mathrm{GeV})$; cf. (9). There we take $a_1^K(1\,\mathrm{GeV}) = 0.05 \pm 0.02$ [26] in our discussion. As for the twist-3 wave function Ψ_p, we take the two groups of parameters as shown in 14 and 15) to do the calculation.

Next, we compare the $\mathcal{O}\left(1/m_b^2\right)$ result of the form factor with the leading order one, which is of order $\mathcal{O}(1/m_b)$ and is calculated by using the WW-like B-meson wave function, and also we make a comparison with the LCSR results of [11, 13] in the large and intermediate energy regions. Through comparison, preferable values for the undetermined parameters can be found. The $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ have been studied within the framework of QCD LCSR [13], especially at $q^2 = 0$; this study shows that

$$F_{+,0}^{B\to K}(0) = 0.331 \pm 0.041 + 0.25\left[a_1^K(1\,\mathrm{GeV}) - 0.17\right]; \tag{16}$$

e.g. when $a_1^K(1\,\mathrm{GeV}) = 0.05$, $F_{+,0}^{B\to K}(0) = 0.301 \pm 0.041$. More generally, $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ can be parameterized in the following form [13]:

$$F_{+,0}^{B\to K}(q^2) = f^{\mathrm{as}}(q^2) + a_1^K(\mu_0) f^{a_1^K}(q^2) + a_2^K(\mu_0) f^{a_2^K}(q^2) + a_4^K(\mu_0) f^{a_4^K}(q^2), \tag{17}$$

where f^{as} contains the contributions to the form factor from the asymptotic DA and all higher-twist effects from 3-particle quark–quark–gluon matrix elements, and $f^{a_1^K, a_2^K, a_4^K}$ contains the contribution from the higher Gegenbauer term of DA that is proportional to a_1^K, a_2^K and a_4^K respectively. Here the factorization scale μ_0 should be taken as $2.2\,\mathrm{GeV}$, since the functions $f^{\mathrm{as}, a_1^K, a_2^K, a_4^K}$ are determined with $\mu_0 = 2.2\,\mathrm{GeV}$ [13]. The explicit expressions of $f^{\mathrm{as}, a_1^K, a_2^K, a_4^K}$ can be found in Table V and Table IX of [13]. For the Gegenbauer moments $a_2^K(2.2\,\mathrm{GeV})$ and $a_4^K(2.2\,\mathrm{GeV})$, we take their preferred values: $a_2^K(2.2\,\mathrm{GeV}) = 0.080$ and $a_4^K(2.2\,\mathrm{GeV}) = -0.0089$ [13]. As regards $a_1^K(2.2\,\mathrm{GeV})$, it equals $0.793 a_1^K(1\,\mathrm{GeV})$, as is seen with the help of QCD evolution.

3.1 Basic properties of the form factor up to $\mathcal{O}(1/m_b^2)$

First, we discuss the properties of $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ caused by the B-meson wave function. For such a purpose, we fix the kaonic wave functions by setting $a_1^K(1\,\mathrm{GeV}) = 0.05$ and by using the Group 1 parameters for Ψ_p. We show the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ with $\delta = \delta_c = 0.275$ in Fig. 2, where $\bar{\Lambda}$ varies within the region $[0.5\,\mathrm{GeV}, 0.6\,\mathrm{GeV}]$.

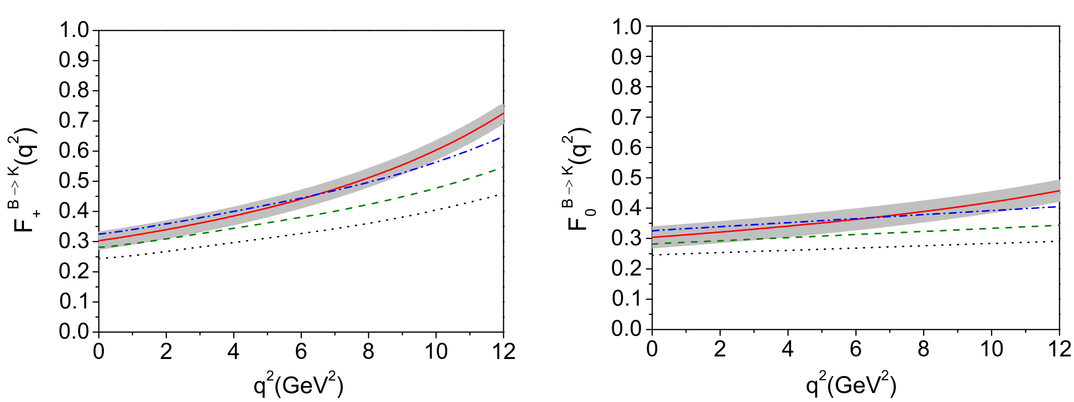


Fig. 2. PQCD results for the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ (*left*) and $F_0^{B\to K}(q^2)$ (*right*) with $\delta = 0.275$ and $a_1^K(1\,\mathrm{GeV}) = 0.05$. The *dash-dot line*, the *dashed line* and the *dotted line* stand for $\bar{\Lambda} = 0.50\,\mathrm{GeV}$, $0.55\,\mathrm{GeV}$ and $0.60\,\mathrm{GeV}$ respectively. For comparison, the *solid line* comes from the QCD LCSR result as shown in (17) and the *fuscous shaded band* shows its theoretical error $\pm 10\%$

For comparison, we show the QCD LCSR result with $a_1^K(1\,\mathrm{GeV}) = 0.05$ and its theoretical error ($\sim \pm 10\%$) [13] by a fuscous shaded band in Fig. 2. The results show that the $B \to K$ transition form factors will decrease with the increment of $\bar{\Lambda}$. The best fit of the QCD LCSR result at $q^2 = 0$ shows that $\bar{\Lambda} \cong \bar{\Lambda}_c = 0.525\,\mathrm{GeV}$. Moreover, we show $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ with $\bar{\Lambda} = \bar{\Lambda}_c = 0.525\,\mathrm{GeV}$ in Fig. 3, where δ varies within the region $[0.25, 0.30]$. The results show that the $B \to K$ transition form factors will increase with the increment of δ. It can be found that when setting $a_1^K(1\,\mathrm{GeV}) = 0.05$, and by varying δ within the region of $[0.25, 0.30]$ and $\bar{\Lambda}$ within the region of $[0.5\,\mathrm{GeV}, 0.6\,\mathrm{GeV}]$, $F_{+,0}^{B\to K}(0)$ runs within the region of $[0.23, 0.34]$. Since the best agreement between the PQCD result and the QCD LCSR result at $q^2 = 0$ is obtained around $\bar{\Lambda}_c = 0.525\,\mathrm{GeV}$ and $\delta_c = 0.275$, we shall always take $\bar{\Lambda} = \bar{\Lambda}_c$ and $\delta = \delta_c$ to do our following calculations if not specially stated.

Second, we discuss the properties of $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ caused by the twist-2 wave function Ψ_K, i.e. by the value of $a_1^K(1\,\mathrm{GeV})$. For such a purpose, we fix the B-meson wave functions by setting $\delta = \delta_c$ and $\bar{\Lambda} = \bar{\Lambda}_c$ and we use the Group 1 parameters for Ψ_p. We show the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ in Fig. 4 with $a_1^K(1\,\mathrm{GeV}) = 0.03$, 0.05 and 0.07 respectively. It can be found that the form factors shall be increased with the increment of $a_1^K(1\,\mathrm{GeV})$, which agrees with the observation of [13]. Furthermore, since the contribution from Ψ_p is sizable compared to that of Ψ_K, it is necessary to discuss its uncertainty in the form factor of the $B \to K$ transition . Figure 5 shows $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ with the two groups of parameters for Ψ_p. The results are very close to each other due to the close shape of their ϕ_p as shown in Fig. 1; e.g., around the region of $q^2 \sim 0$ the difference between them is less than 6%. So by taking the proper transverse momentum dependence for the wave function Ψ_p, where we have taken the BHL prescription for its transverse momentum dependence, the uncertainties from its distribution amplitude ϕ_p can be reduced.

Finally, in order to get a deep understanding of the $B \to K$ transition form factor, we discuss the contributions from different parts of the B-meson wave function or the kaon wave function, respectively. Here we take $F_+^{B\to K}(q^2)$ in our discussion; the case of $F_0^{B\to K}(q^2)$ can be done in a similar way. For convenience, we set $\bar{\Lambda} = \bar{\Lambda}_c$, $\delta = \delta_c$ and $a_1^K(1\,\mathrm{GeV}) = 0.05$ and we use the Group 1 parameters for Ψ_p. When discussing the contribution from one of the kaon wave function structures, the contribution from all

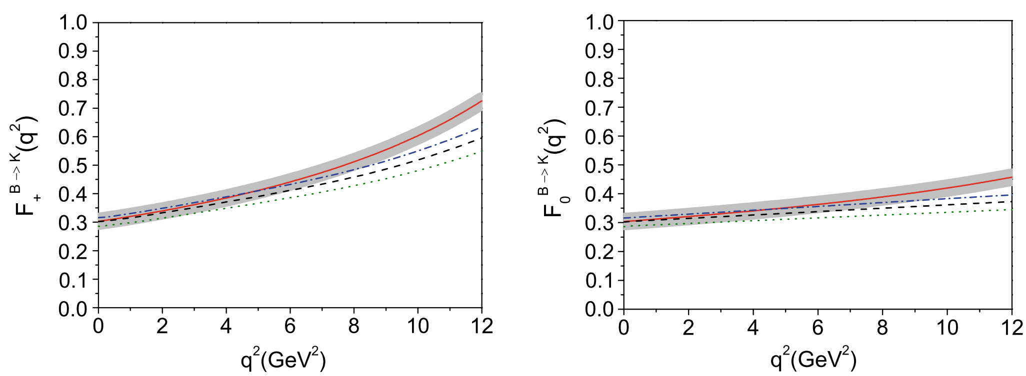


Fig. 3. PQCD results for the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ (*left*) and $F_0^{B\to K}(q^2)$ (*right*) with $\bar{\Lambda} = 0.525$ GeV and $a_1^K(1\,\mathrm{GeV}) = 0.05$. The *dotted line*, the *dashed line* and the *dash-dot line* stand for $\delta = 0.25$, 0.275 and 0.30 respectively. For comparison, the *solid line* comes from the QCD LCSR as shown in (17) and the *fuscous shaded band* shows its theoretical error $\pm 10\%$

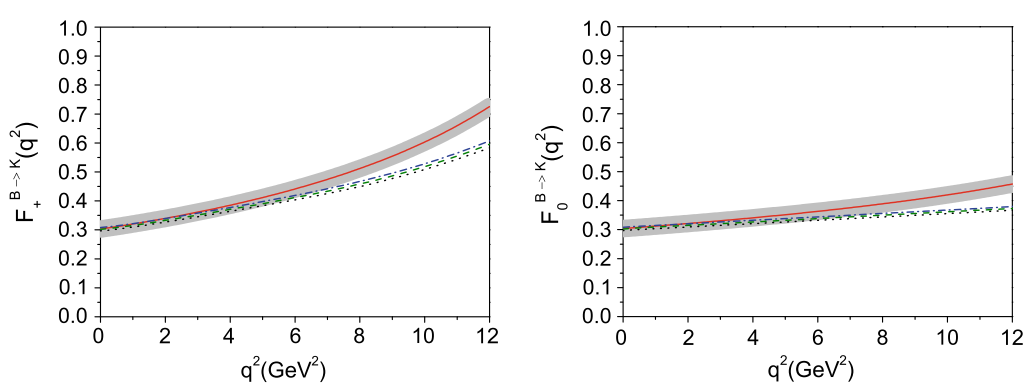


Fig. 4. PQCD results for the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ (*left*) and $F_0^{B\to K}(q^2)$ (*right*) with $\bar{\Lambda} = 0.525$ GeV and $\delta = 0.275$. The *dotted line*, the *dashed line* and the *dash-dot line* stand for $a_1^K(1\,\mathrm{GeV}) = 0.03$, 0.05 and 0.07 respectively

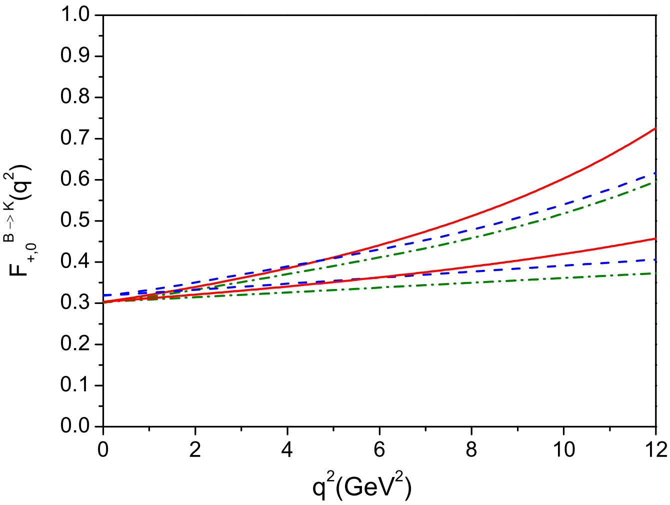
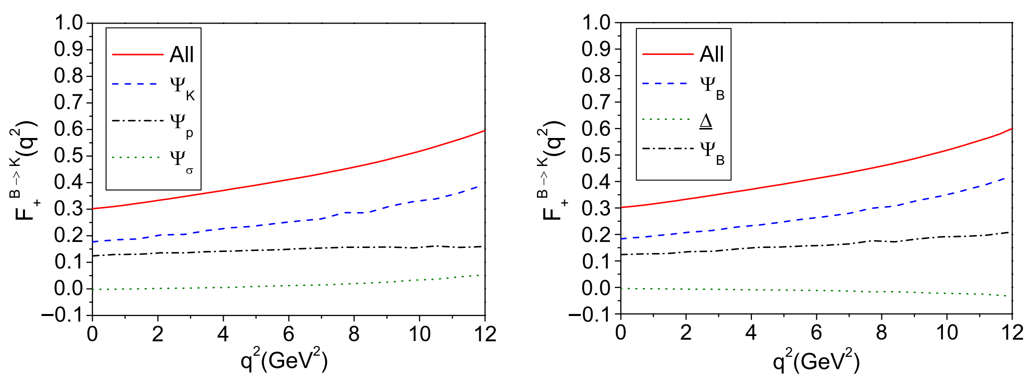


Fig. 5. PQCD results for the $B \to K$ transition form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$ with $\bar{\Lambda} = 0.525$ GeV, $\delta = 0.275$ and $a_1^K(1\,\mathrm{GeV}) = 0.05$. The *dash-dot* and the *dashed lines* are for Ψ_p with Group 1 parameters (14), Group 2 parameters, see (15), respectively. For comparison, the *solid lines* come from the QCD LCSR with $a_1^K(1\,\mathrm{GeV}) = 0.05$ [13]

the B-meson wave function structures are summed up, and vice versa. Figure 6a shows the contributions from the different twist structures of the kaon wave function, i.e. Ψ_K, Ψ_p and Ψ_σ (the contributions from the terms involving Ψ'_σ are included in Ψ_σ), respectively. One may observe that the contribution from Ψ_p is comparable to that of Ψ_K; e.g., its contribution is about 70% of that of Ψ_K at $q^2 \simeq 0$, and the contribution from Ψ_σ is small. Figure 6b presents the contributions from Ψ_B, $\bar{\Psi}_B$ and Δ respectively. It can be found that the contribution of $\bar{\Psi}_B$ is about $50\% - -67\%$ of that of Ψ_B in the region of $q^2 \in [0, 10\,\mathrm{GeV}^2]$, while the contribution from Δ is negligible in comparison to that of Ψ_B and $\bar{\Psi}_B$. So the contribution from $\bar{\Psi}_B$ should be included for a consistent estimation to the next leading order. As a comparison, it can be found that under the leading-order estimation the contribution from $\bar{\Psi}_B$ is only about 20% of that of Ψ_B at $q^2 = 0$, which agrees with the rough order estimation that the contribution from $\bar{\Psi}_B$ is of order $\mathcal{O}(1/m_b)$. So for the leading order estimation, $\mathcal{O}(1/m_b)$, the contribution from $\bar{\Psi}_B$ is usually neglected in the literature. Such a difference of the contribution of $\bar{\Psi}_B$ of the leading order estimation and the next-to-leading order estimation is mainly due to the fact that the transverse momentum dependence of the B-meson wave functions is merely a delta function under the WW approximation (the leading-order estimation), while it shall be broadened to a certain degree according to the value of δ by taking into account the 3-particle Fock states' contributions (the next-to-leading order estimation); cf. Fig. 2 of [2]. So, qualitatively the contributions from $\bar{\Psi}_B$ shall be increased to a certain degree for the next-to-leading order case, due to the lesser suppression of the endpoint region ($\xi \to 0$) from the transverse momentum distributions, compared to that of the leading order case. Then the naive order estimation for the contribution of $\bar{\Psi}_B$ is no longer correct, and the contributions from Ψ_B and $\bar{\Psi}_B$ are both important in the next-to-leading order calculation.

3.2 Comparison with the leading order results

The WW-like B-meson wave functions in the compact parameter b_B-space can be found in [1]. Taking the WW-like wave functions and cutting off the power suppressed terms in the hard scattering amplitude, we can obtain the leading order results ($\mathcal{O}(1/m_b)$) for the form factors $F_+^{B\to K}(q^2)$ and $F_0^{B\to K}(q^2)$. Strictly, one should cut off the contribution from $\bar{\Psi}_B$ to obtain the leading order estimation, since $\bar{\Psi}_B$ is power suppressed in comparison to Ψ_B. However, for easy comparison with the results in the literature, e.g. [10], we keep $\bar{\Psi}_B$ in the leading order estimation. For convenience, we take $\bar{\Lambda} = \bar{\Lambda}_c$, $\delta = \delta_c$ and $a_1^K(1\,\mathrm{GeV}) = 0.05$ and we use the Group 1 parameters for the wave function Ψ_p to make a comparison of the leading order results



(a) (b)

Fig. 6. PQCD results for the $B \to K$ transition form factor $F_+^{B\to K}(q^2)$ with fixed $\bar{\Lambda} = 0.55$ GeV, $\delta = 0.275$ and $a_1^K(1\,\mathrm{GeV}) = 0.05$. The *left diagram* is for the different kaon twist structures, Ψ_K, Ψ_p and Ψ_σ. The *right diagram* is for the different B-meson structures, Ψ_B, $\bar{\Psi}_B$ and Δ

with the total results that include the contributions up to order $\mathcal{O}\left(1/m_b^2\right)$. It can be found that the leading order results are smaller than the total results by about 25% in the large recoil region; e.g., at $q^2 = 0$, the leading order $F_{+,0}^{B\to K}(0) = 0.229$. One may observe that a larger leading order estimation has been obtained in [10], which shows that $F_{+,0}^{B\to K}(0) = 0.321 \pm 0.036$. We argue that the present leading order estimation is more reliable, and the larger value of $F_{+,0}^{B\to K}(0)$ derived in [10] is mainly due to the following two reasons. 1) Even though the Sudakov and threshold resummation factors will kill the endpoint singularity of the process [6, 7, 10, 22, 23], the transverse momentum dependence of kaonic wave functions is still important to give a more reliable PQCD estimation, which is similar to the cases of the $B \to \pi$ form factor [1] and the electromagnetic form factor of the pion [34, 35]. 2) The distribution amplitude of Ψ_K with a much bigger value of $a_1^K(1\,\mathrm{GeV})$, i.e. $a_1^K(1\,\mathrm{GeV}) = 0.17$, is adopted by [10]. As to our first item, in [10] the transverse momentum dependence of kaonic wave functions is lacking, i.e. a distribution amplitude different from the wave function is used. On the other hand, in our present calculation, the BHL prescription is adopted for the kaonic transverse momentum dependence. As for the wave function $\Psi_p(x, \mathbf{k}_\perp)$, such a transverse momentum dependence will result in a double humped DA ϕ_p, as shown in Fig. 1, and this will give a more effective suppression in the endpoint region than the one used in [10]. In fact, it can be found that the contributions from the endpoint region shall always be overestimated without taking the transverse momentum into the twist-3 wave function $\Psi_p(x, \mathbf{k}_\perp)$[3]. Furthermore, by taking the proper transverse momentum dependence for the wave function Ψ_p, the uncertainties from its distribution amplitude ϕ_p can be reduced as has been discussed in Sect. 3.1.

As we stated as our second item, the distribution amplitude of Ψ_K with a much bigger value of $a_1^K(1\,\mathrm{GeV})$, i.e. $a_1^K(1\,\mathrm{GeV}) = 0.17$, is adopted by [10]. Since the form factors increases with the increment of $a_1^K(1\,\mathrm{GeV})$, a larger value of $a_1^K(1\,\mathrm{GeV})$ shall increase the form factors.

Furthermore, by varying $\bar{\Lambda}$ within the region of $[0.50, 0.55]$, the uncertainty caused by $\bar{\Lambda}$ is the biggest one, being of order $(1/m_b)$. By varying δ within the region of $[0.25, 0.30]$, the uncertainty caused by δ is smaller, being of order $\left(1/m_b^2\right)$. This can be qualitatively explained by the fact that $\bar{\Lambda}$ is the characteristic parameter that determines the leading Fock state behavior of the B-meson wave functions, while δ is the characteristic parameter that determines the higher Fock state behavior of the B-meson wave functions. The uncertainties from a_1^K and Ψ_K are less than 10% in the large recoil region.

3.3 Comparison with the LCSR results

The $B \to K$ transition form factor has been analyzed by several groups in the QCD LCSR approach [11–14]. A new sum rule for $B \to K$ is derived from the correlation functions expanded near the light cone in terms of the B-meson distributions [11], in which the contributions of the quark–antiquark and quark–antiquark–gluon components in the B-meson are taken into account. It has been found that the $B \to K$ transition form factor in the large recoil region does not receive contributions from the 3-particle B-meson DAs. One may observe that if substituting the B-meson DAs, which are derived by doing the integration over b_B in 4 and 5 into the formulae of [11], then one can obtain the same results as [11], since our B-meson DAs are close to the exponential model wave functions adopted in [11]. Furthermore, one may observe that the result of $F_{+,0}^{B\to K}(0) = 0.31 \pm 0.04$ under the condition of $a_1^K(1\,\mathrm{GeV}) = 0.05 \pm 0.03$ agrees well with our present PQCD estimation. Secondly, a systematic QCD LCSR calculation has been done in [13] by including the one-loop radiative corrections to the twist-2 and twist-3 contributions, and leading-order twist-4 corrections. Some comparison of their results with our present one can be found in Figs. 2, 3 and 4, which also shows good agreement within reasonable errors. For example, from (17) it can be found that the uncertainty of the form factor caused by $a_1^K(1\,\mathrm{GeV})$ within the region of $[0.03, 0.07]$ is less than 5%, which is consistent with our present result as shown in Fig. 4.

4 Discussion and summary

In this paper, we have examined the form factor of the $B \to K$ transition in the PQCD approach up to order $\mathcal{O}\left(1/m_b^2\right)$, where the transverse momentum dependence for the wave function, the Sudakov effects and the threshold effects are included to regulate the endpoint singularity and to derive a more reasonable result. We have confirmed that the PQCD approach can be applied to a calculation of the $B \to K$ transition form factor in the large recoil regions. We emphasize that the transverse momentum dependence for both the B-meson and the kaon is important to give a better understanding of the $B \to K$ transition form factor. Figure 6a shows that the contribution from the pionic twist-3 wave function Ψ_p is sizable in comparison to that of Ψ_K, and the contribution from Ψ_σ is small. Furthermore, Fig. 6b shows that by using the B-meson wave functions up to next-to-leading order in a Fock state expansion, the contributions from Ψ_B and $\bar{\Psi}_B$ are important.

In [1, 2], we have shown that the results from the PQCD approach, the lattice QCD approach and the QCD LCSRs are complementary, and by combining the results of those three approaches, one can obtain an understanding of the $B \to \pi$ transition form factor in the whole physical region. Also, the best fit of the PQCD results with that of the QCD LCSR results in the large recoil region can be obtained by taking $\bar{\Lambda} \in [0.50, 0.60]$ and $\delta \in [0.25, 0.30]$ [2]. In the present paper, we show that within the regions of $\bar{\Lambda} \in [0.50, 0.55]$, $\delta \in [0.25, 0.30]$ and $a_1^K(1\,\mathrm{GeV}) \in [0.03, 0.07]$, the PQCD results on the $B \to K$ form factor in the large recoil region also agree well with the QCD LCSR results [11, 13].

[3] For example, a detailed discussion of the model dependence of the pionic twist-3 wave function $\Psi_p(x, \mathbf{k}_\perp)$ can be found in [34, 35].

Our present PQCD results in some sense are more reliable than the LCSR calculations due to the fact that by taking the transverse momentum dependence properly for the wave functions the soft endpoint singularity has been effectively suppressed; e.g., as is shown in Sect. 3.1 the difference of the two models for Ψ_p is less than 6% in the large recoil region, while for the LCSR approach a large uncertainty comes from the kaonic twist-3 DA ϕ_p, which is not too well known. By running the parameters within the above regions, we obtain $F_{+,0}^{B\to K}(0) = 0.30 \pm 0.04$. Finally, to illustrate the $\mathrm{SU}_f(3)$-breaking effects, we calculated the ratio with the help of the $B \to \pi$ results in [2]: $[F_{+,0}^{B\to K}(0)/F_{+,0}^{B\to \pi}(0)] = 1.13 \pm 0.02$, which favors small $\mathrm{SU}_f(3)$-breaking effects.

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